Lesson 18. Stochastic Dynamic Programming, cont.

1 The problem

Suppose you have \$5,000 to invest. Over the next 3 years, you want to double your money. At the beginning of each of the next 3 years, you have an opportunity to invest in one of two investments: A or B. Both investments have uncertain profits. For an investment of \$5,000, the profits are as follows:

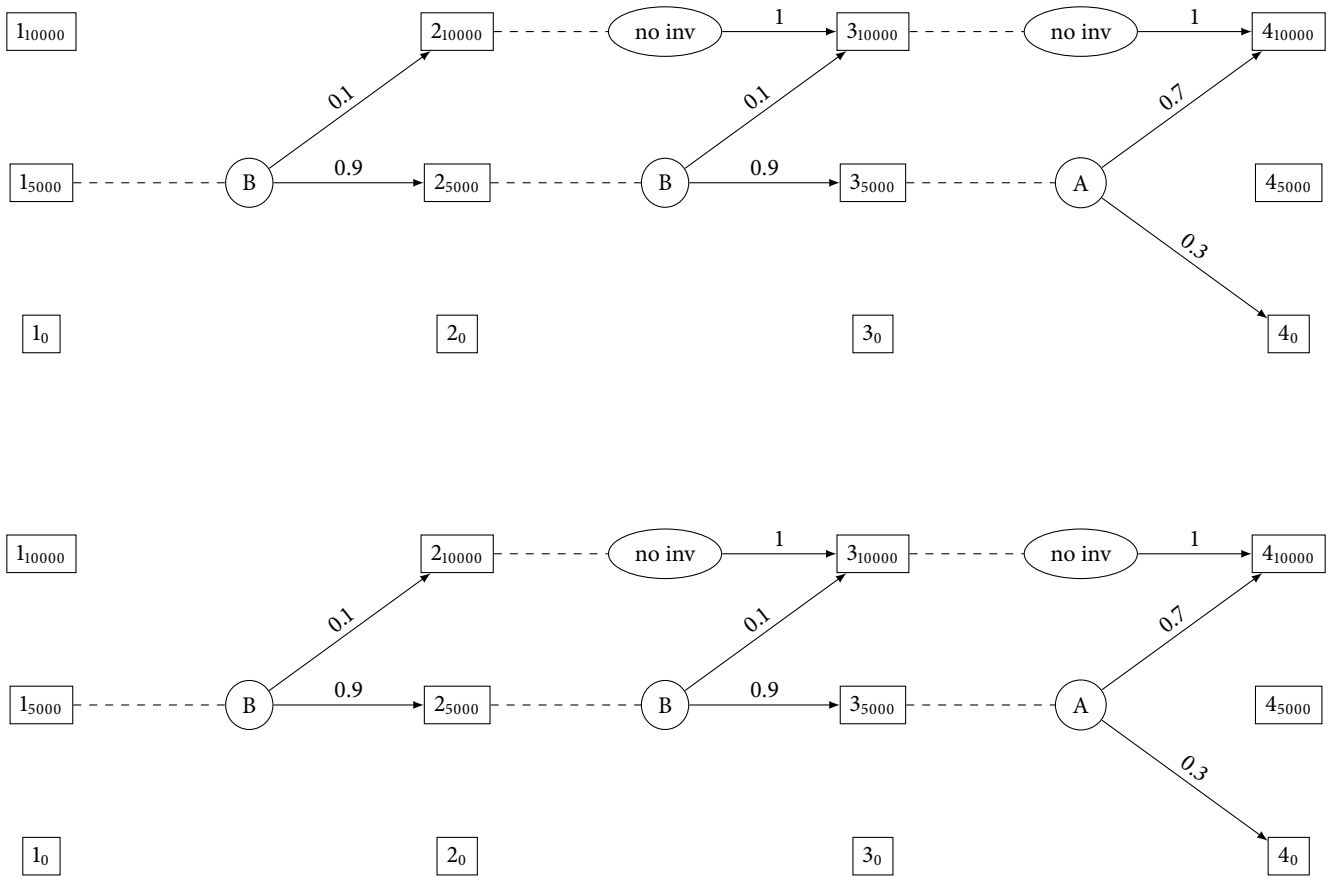
Investment	Profit (\$)	Probability
A	−5,000	0.3
	5,000	0.7
B	0	0.9
	5,000	0.1

You are allowed to make at most one investment each year, and can invest only \$5,000 each time. Any additional money accumulated is left idle. Once you've accumulated \$10,000, you stop investing.

Formulate a stochastic dynamic program to find an investment policy that maximizes the probability you will have \$10,000 after 3 years.

2 Warm up

Consider the following investment policy. What is the probability of having at least \$10,000?



1_{10000}
2_{10000}
no inv
1
3_{10000}
no inv
1
4_{10000}
0.1
0.1
0.7
1_{5000}
B
0.9
2_{5000}
B
0.9
3_{5000}
A
4_{5000}
0.3
1_0
2_0
3_0
4_0

3 Formulating the stochastic dynamic program

- Stages:

- States:

- Allowable decisions x_t at stage t and state n:

- Sketch of basic structure – transition probabilities and contributions:

- In words, the value-to-go $f_t(n)$ at stage t and state n is:

- Value-to-go recursion

$$f_t(n) = \min_{x_t \text{ allowable}} / \max \left\{ \sum_{m \text{ state}} p(m \mid n, t, x_t) \Big[c(m \mid n, t, x_t) + f_{t+1}(m) \Big] \right\} \quad \text{for stages } t \text{ and states } n$$

- Boundary conditions:

- Desired value-to-go function value:

4 Interpreting the value-to-go function

- Solving the recursion on $f_t(n)$, we obtain:

t	n	$f_t(n)$	x_t^*
1	0	0	no investment
1	5000	0.757	B
1	10000	1	no investment
2	0	0	no investment
2	5000	0.73	B
2	10000	1	no investment
3	0	0	no investment
3	5000	0.7	A
3	10000	1	no investment

- Based on this, what should your investment policy be?

- What is your probability of having \$10,000?